SPM ADDITIONAL MATHEMATICS KBAT COLLECTION MARKING SCHEME

MARKING GUIDE

Note: I am by no means able to produce an accurate marking scheme that follows LPM marking scheme. This is just what I deduced the marking scheme might be based on the marking schemes I've seen in SPM trial papers and my Pre-U papers. So take it with a grain of salt.

In general, SPM Additional Mathematics paper only have two types of marks, K and N. Where K represents Kerja (Working) and N represents Nilai (Value).

This marking scheme that I made goes a little beyond that. Having four different types of marks allocated.

i. K (Kerja) : Given when the student that attempts to solve the problem using the correct approach, such as factorisation and substitution.

ii. J (Jawapan) : Given when the student obtained the correct value for the unknown, or expressed the unknown in terms of other unknown successfully.

iii. L (Lukisan) : Given when the students successfully drawn/sketched the diagram and labelled the critical parts of the diagram correctly.

iv. B (Bentuk) : Given when the students used a correct mathematical concepts to solve the problem, such as making statements, deductions or sound arguments.

The terms such as K1 or J1 (highlighted in bold) will be put next to the line where it would be given the marks on. With the letters being the types of marks given, and the number being the amount of marks given (rarely will be different from 1) for that line of working. Note that if you did skip that line of working that specifically has a term next to it, but you wrote the line that is the next step from that particular line, or you used a different method for solving (different substitution pathways), you will still be able to get the marks.

It is important to know that most of the time, getting a correct answer with wrong steps will give you K0 and J0 (no marks) while getting a wrong answer with steps that were right may yield some K1 marks with J0. This is why teachers tell you to write all of the necessary steps. Equivalent answers will get marks.

Some marks will be given with specific conditions, such as factorisation and substitutions. These marks will have a remark next to them in the scheme, if the student did them correctly, they will get the mark. Some common ones includes

- Factorisation : Given when the student correctly factorised the given expression
- Substitution : Given when the student correctly substituted the (correct) equation that they found from the previous steps.
- Seen : Given when the specfic term is seen in the line.

Some marks will be allocated to two different steps, and will have an arrow that point towards both of them to indicate which line is the mark given to.

- Either : If one of the two highlighted lines is correct, mark is given.
- Both : Marks is only given if both highlighted lines are correct.

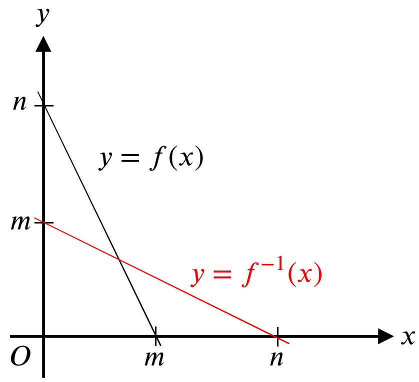
FUNCTIONS

1 (a)

$f(m) = 0$

$f^{-1}(0) = m$ **J1**

(b)



Correct Line - L1

Total: 2

2 (a)

$$f(x) = \frac{ax+b}{cx+d}$$

$$f^{-1}\left(\frac{ax+b}{cx+d}\right) = x$$

Let $y = \dfrac{ax+b}{cx+d}$

$cxy + dy = ax + b$ **K1**

$cxy - ax = b - dy$

$x(cy - a) = (b - dy)$

$$x = \frac{b-dy}{cy-a}$$

$$f^{-1}(y) = \frac{b-dy}{cy-a}$$

$f^{-1}(x) = \dfrac{b-dx}{cx-a}, x \neq \dfrac{a}{c}$ **J1**

(b) i) According to (a),

$g^{-1}(x) = \dfrac{8+5x}{x-1}, x \neq 1$ **J1**

ii) According to (a),

$h^{-1}(x) = \dfrac{-3-4x}{x-2}, x \neq 2$ **J1**

(c)

$f(x) = f^{-1}(x)$

$$\frac{ax+b}{cx+d} = \frac{b-dx}{cx-a}$$

$$\frac{ax+b}{cx+d} = \frac{-dx+b}{cx-a}$$

$a = -d$ **J1**

Total: 5

3	$$ff(x) = f\left(\frac{m}{x} + n\right)$$ $$= \frac{m}{\left(\frac{m}{x} + n\right)} + n$$ $$= \frac{mx}{m + nx} + n \quad \textbf{J1}$$ $$f(x) = \frac{m}{x} + n$$ $$f^{-1}\left(\frac{m}{x} + n\right) = x$$ Let $y = \frac{m}{x} + n$ $$y - n = \frac{m}{x}$$ $$\frac{m}{y-n} = x$$ $$f^{-1}(y) = \frac{m}{y-n}$$ $$f^{-1}(x) = \frac{m}{x-n} \quad \textbf{J1}$$ $$ff(x) = f^{-1}(x)$$ $$\frac{mx}{m+nx} + n = \frac{m}{x-n}$$ $$mx(x-n) + n(x-n)(m+nx) = m(m+nx) \quad \textbf{K1}$$ $$mx^2 - mnx + mnx + n^2x^2 - mn^2 - n^3x = m^2 + mnx$$ $$mx^2 + n^2x^2 - n^3x - mnx - mn^2 - m^2 = 0$$ $$\left(m + n^2\right)x^2 + \left(-n^3 - mn\right)x - mn^2 - m^2 = 0$$ $$\left(m+n^2\right)x^2 - n\left(m+n^2\right)x - m\left(m+n^2\right) = 0 \quad \textbf{Factorisation - K1}$$ $$\left(m+n^2\right)\left(x^2 - nx - m\right) = 0$$ Since $x^2 - nx - m \neq 0.\, m + n^2 = 0.$ **J1**			
4	(a)	i)	$$f^2(x) = ff(x)$$ $$= f\left(\frac{x-1}{x+1}\right)$$ $$= \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1}$$ $$= \frac{x - 1 - (x+1)}{x - 1 + (x+1)}$$ $$= -\frac{2}{2x}$$ $$= -\frac{1}{x} \quad \textbf{J1}$$
		ii)	$$f^4(x) = ffff(x)$$ $$= f^2\left[f^2(x)\right] \quad \textbf{B1}$$ $$= f^2\left(-\frac{1}{x}\right)$$ $$= -\frac{1}{-\frac{1}{x}}$$ $$= x \quad \textbf{J1}$$

	(*b*)		$f^2(x) = -\frac{1}{x}$ $f^4(x) = x$ $f^6(x) = -\frac{1}{x}$ $f^8(x) = x$ $f^{10}(x) = -\frac{1}{x}$ $f^{12}(x) = x$ **At least four - K1** Hence, $f^n(x) = -\frac{1}{x}$, if n is even but not divisible by 4. $f^n(x) = x$, if n is divisible by 4. $\therefore f^{26}(x) = -\frac{1}{x}$ **J1** **Total: 5**
5	(*a*)	i)	$g(x) = \frac{2x-5}{3x-p}$ $g^{-1}\left(\frac{2x-5}{3x-p}\right) = x$ Let $y = \frac{2x-5}{3x-p}$ $3xy - py = 2x - 5$ **K1** $3xy - 2x = py - 5$ $x(3y-2) = py - 5$ $x = \frac{py-5}{3y-2}$ $g^{-1}(y) = \frac{py-5}{3y-2}$ $g^{-1}(x) = \frac{px-5}{3x-2}$ **J1**
		ii)	$g^{n-1}(x) = \frac{px-5}{3x-2}$ $g^{n-1}(x) = g^{-1}(x)$ $n - 1 = -1$ $n = 0$ **J1**
	(*b*)		$g(x) = g^{-1}(x)$ $\frac{2x-5}{3x-p} = \frac{px-5}{3x-2}$ $p = 2$ **J1** $3q - p = 0$ $3q - 2 = 0$ $q = \frac{2}{3}$ **J1** **Total: 5**

6	(*a*)	$f(x)$ is a quadratic function, therefore for $f(x)$ to have an inverse, $x = p$ must be the axis of symmetry. $p = -\frac{b}{2a}$ $= -\frac{-11}{2(1)}$ $= \frac{11}{2}$ **J1**
	(*b*)	Since p is the axis of symmetry, $f(p)$ will be its minimum point, there will be no intersection between $y = a$ and the curve as $y = a$ is below the minimum point. $\therefore$ Number of solutions $= 0$ **J1** **Total: 2**
7		If $g(x)$ does not have an inverse, it is a many-to-one function. Which implies that $g(a) = g(b)$ when $a \neq b$. $g(a) = g(b)$ **B1** $\frac{ak+8}{4a-5} = \frac{bk+8}{4b-5}$ $(ak+8)(4b-5) = (bk+8)(4a-5)$ $4abk - 5ak + 32b - 40 = 4abk - 5bk + 32a - 40$ **K1** $32b - 5ak = 32a - 5bk$ $32a + 5ak - 32b - 5bk = 0$ $a(32+5k) - b(32+5k) = 0$ **Factorisation - K1** $(a-b)(32+5k) = 0$ Since $a \neq b, a - b \neq 0$. Thus $32 + 5k = 0$ **B1** $k = -\frac{32}{5}$ **J1** **Total: 5**
8	(*a*)	Let $g(x)$ be the price of item from the retailer; $g(x) = 6x + 35$ **J1**
	(*b*)	$P = 2x + 10$ $x = \frac{P-10}{2}$ $g(P) = \frac{6(P-10)}{2} + 35$ **K1** $= 3P - 30 + 35$ $= 3P + 5$ **J1**
	(*c*)	$g(x) > P$ $6x + 35 > 2x + 10$ $4x > -25$ $x > -6.25$ **K1** $\therefore x > 0$ **J1** **Total: 5**

QUADRATIC FUNCTIONS

1

$$\alpha - 2\beta = 0$$
$$\alpha = 2\beta$$

$$2x^2 = ax - b$$
$$2x^2 - ax + b = 0$$

$$\text{SOR} = \alpha + \beta$$
$$-\frac{-a}{2} = \alpha + \beta$$
$$\frac{a}{2} = 2\beta + \beta$$ **Substitution - K1**
$$\frac{a}{2} = 3\beta$$
$$\beta = \frac{a}{6}$$

Either - K1

$$\text{POR} = \alpha\beta$$
$$\frac{b}{2} = \alpha\beta$$
$$\frac{b}{2} = (2\beta)\beta$$
$$\frac{b}{2} = 2\beta^2$$
$$\frac{b}{4} = \left(\frac{a}{6}\right)^2$$ **Substitution - K1**
$$\frac{a^2}{36} = \frac{b}{4}$$
$$a^2 = 9b$$
$$a = \pm 3\sqrt{b}$$ **J1**

Total: 4

2

$$x^2 + px - \frac{pq}{2} = qx$$
$$x^2 + px - qx - \frac{pq}{2} = 0$$
$$x^2 + (p - q)x - \frac{pq}{2} = 0$$

$$b^2 - 4ac = (p - q)^2 - 4(1)\left(-\frac{pq}{2}\right)$$ **K1**
$$= p^2 - 2pq + q^2 + 2pq$$
$$= p^2 + q^2$$ **J1**

Since both p^2 and q^2 are both non-negative real numbers (≥ 0), for all values of p and q, their sum must also be non-negative real numbers (≥ 0) . **B1**

The discriminant is a non-negative real number (≥ 0), thus the equation will have real roots for all values of p and q. **B1**

Total: 4

3	Let $f(x) = -x^2 + 2x - nx + 16$ $= -x^2 + (2-n)x + 16$ $2 - n = 0$ $n = 2$ **J1** $f(x) = -x^2 + (2-2)x + 16$ $= -x^2 + 16$ $= -\left(x^2 - 16\right)$ $= -(x-4)(x+4)$ **K1** Roots for $f(x)$: $x = 4$ and $x = -4$ Let $g(x) = x^2 - 3k$ $g(4) = 0$ **B1** $4^2 - 3k = 0$ $k = \frac{16}{3}$ **J1** **Total: 4**	
4	(*a*)	**Approach 1** As $p < 0$, the original shape of the curve (before modulus and negative applies) has a maximum point. Hence, $q = 5$. **J1** At $(4, 0)$; $-\left\|p(3-4)^2 + 5\right\| = 0$ $p(-1)^2 = -5$ $p = -5$ **J1** **Approach 2** At $(3, -5)$; $-\left\|p(3-3)^2 + q\right\| = -5$ $\|q\| = 5$ $q = \pm 5$ **K1** At $(4, 0)$; When $q = -5$, $-\left\|p(4-3)^2 - 5\right\| = 0$ $p = 5$ (rejected) When $q = 5$ $-\left\|p(4-3)^2 + 5\right\| = 0$ $p = -5$ **Both - J1**
	(*b*)	When $x = 6$, $y = -\left\|-5(6-3)^2 + 5\right\|$ $= -40$ **K1** Range: $-40 \leq y \leq 0$ **J1** **Total: 4**

5	Let the speed of the boat $= x$ $$\frac{\text{Distance}}{\text{Speed}} = \text{Time}$$ $\frac{24}{x+3} + \frac{24}{x-3} = 6$ **B1** $24(x-3) + 24(x+3) = 6(x-3)(x+3)$ **K1** $24x - 72 + 24x + 72 = 6x^2 - 54$ $6x^2 - 48x - 54 = 0$ $x^2 - 8x - 9 = 0$ $(x-9)(x+1) = 0$ **Factorisation - K1** $x = 9, x = -1$ (rejected) $\therefore x = 9$ **J1** **Total: 4**	
6	(*a*)	At $(0, 5)$; $h(0) = 5$ $a(0)^2 + \frac{1}{2}(0) + c = 5$ $c = 5$ **J1** At $(200, 5)$; $h(200) = 5$ $a(200)^2 + \frac{1}{2}(200) + 5 = 5$ $40000a = -100$ $a = -\frac{1}{400}$ **J1** Max height at $x = 100$; $h(100) = -\frac{1}{400}(100)^2 + \frac{1}{2}(100) + 5$ **B1** $= -25 + 50 + 5$ $= 30$ **J1**
	(*b*)	Let the distance be m; $h(m) = 20$ $-\frac{1}{400}m^2 + \frac{1}{2}m + 5 = 20$ **B1** $m^2 - 200m - 2000 = -8000$ $m^2 - 200m + 6000 = 0$ $m = \frac{-(-200) \pm \sqrt{(-200)^2 - 4(1)(6000)}}{2(1)}$ **K1** $m = \frac{200 \pm \sqrt{16000}}{2}$ $\therefore m = 36.75, m = 163.25$ **Both - J1** **Total: 7**

7	$x + y = 4$ $y = 4 - x$ $y \geq 2$ $4 - x \geq 2$ $2 \geq x$ **B1** Range of values of $x: 0 \leq x \leq 2$ $f(x, y) = 3x^2 + 2y^2$ $f(x) = 3x^2 + 2(4 - x)^2$ **Substitution - K1** $= 3x^2 + 32 - 16x + 2x^2$ $= 5x^2 - 16x + 32$ $= 5\left(x^2 - \frac{16}{5}\right) + 32$ $= 5\left[\left(x - \frac{8}{5}\right)^2 - \left(\frac{8}{5}\right)^2\right] + 32$ $= 5\left(x - \frac{8}{5}\right)^2 + \frac{96}{5}$ **K1** $f(0) = 3(0)^2 + 2(4 - 0)^2$ $= 32$ As $\frac{8}{5}$ is further from 0 than from 2, calculation for $f(2)$ is not required. $\therefore$ Minimum value $= \frac{96}{5}$ **J1** $\therefore$ Maximum value $= 32$ **J1** **Total: 5**	
8	(a)	$\alpha^2 = 3\alpha - 4$ **B1** $\alpha^3 = \alpha^2(\alpha)$ $= (3\alpha - 4)\alpha$ $= 3\alpha^2 - 4\alpha$ $= 3(3\alpha - 4) - 4\alpha$ $= 9\alpha - 12 - 4\alpha$ $= 5\alpha - 12$ **J1**
	(b)	According to (a), $\beta^3 = 5\beta - 12$ **J1**

	(c)	$x^2 = 3x - 4$ $x^2 - 3x + 4 = 0$ SOR $= \alpha + \beta$ $= -\left(-\frac{3}{1}\right)$ $= 3$ POR $= \alpha\beta$ $= \frac{4}{1}$ $= 4$ **Either - J1** $\alpha^3 + \beta^3 = 5\alpha - 12 + 5\beta - 12$ $= 5(\alpha + \beta) - 24$ $= 5(3) - 24$ $= -9$ **J1** New SOR $= \frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}$ $= \frac{\alpha^3 + \beta^3}{\alpha^2\beta^2}$ $= -\frac{9}{(\alpha\beta)^2}$ $= -\frac{9}{4^2}$ $= -\frac{9}{16}$ **J1** New POR $= \left(\frac{\alpha}{\beta^2}\right)\left(\frac{\beta}{\alpha^2}\right)$ $= \frac{1}{\alpha\beta}$ $= \frac{1}{4}$ **J1** New equation: $x^2 - \left(-\frac{9}{16}\right)x + \frac{1}{4} = 0$ $x^2 + \frac{9}{16}x + \frac{1}{4} = 0$ $16x^2 + 9x + 4 = 0$ **J1** **Total: 8**
9		$\alpha^2 - 6\alpha - 2 = 0$ **Either - B1** $\beta^2 - 6\beta - 2 = 0$ $\alpha^2 - 2 = 6\alpha$ $\beta^2 - 2 = 6\beta$ $\frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^{10} - \beta^{10} - 2\alpha^8 + 2\beta^8}{2(\alpha^9 - \beta^9)}$ $= \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$ **Factorisation - K1** $= \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{2(\alpha^9 - \beta^9)}$ $= \frac{6(\alpha^9 - \beta^9)}{2(\alpha^9 - \beta^9)}$ $= 3$ **J1** **Total: 3**

10	(*a*)	**Approach 1** $200 = -\frac{b}{2a}$ $200 = -\frac{b}{2\left(\frac{1}{200}\right)}$ **B1** $b = -2$ **J1** **Approach 2** $h(400) = c$ $\frac{1}{200}(400)^2 + b(400) + c = c$ **B1** $b = -2$ **J1**
	(*b*)	$h(200) = 75$ $\frac{1}{200}(200)^2 - 2(200) + c = 75$ $c = 275$ **J1**
	(*c*)	i) (300, 145) **J1**
		ii) Let the equation be $h(x) = ax^2 + bx + 275$ $h(100) = 145$ $a(100)^2 + b(100) + 275 = 145$ **Seen c = 275 - B1** $a(100)^2 + b(100) = -130$ $100a + b = -1.3$ $b = -1.3 - 100a$ $h(300) = 145$ $a(300)^2 + b(300) + 275 = 145$ $a(300)^2 + b(300) = -130$ $300a + b = -\frac{13}{30}$ $300a - 1.3 - 100a = -\frac{13}{30}$ **Substitution - K1** $200a = \frac{13}{15}$ $a = \frac{13}{3000}$ $b = -1.3 - 100\left(\frac{13}{3000}\right)$ **Either - J1** $= -\frac{26}{15}$ $\therefore h(x) = \frac{13}{3000}x^2 - \frac{26}{15}x + 275$ **J1** **Total: 8**

SYSTEM OF EQUATIONS

1

By Substitution

$x - 2y = 4$
$x = 4 + 2y$

$2x - 3y + 2z = -2$
$2(4 + 2y) - 3y + 2z = -2$
$8 + 4y - 3y + 2z = -2$
$y + 2z = -10$ **K1**
$y = -10 - 2z$

$4x - 7y + 2z = 6$
$4(4 + 2y) - 7y + 2z = 6$
$16 + 8y - 7y + 2z = 6$
$y + 2z = -10$ **K1**
$-10 - 2z + 2z = -10$
$0 = 0$ **J1**

$\therefore$ The system of equations have infinitely many solutions. **B1**

By Elimination

$x - 2y = 4$ ——— (1)
$2x - 3y + 2z = -2$ ——— (2)
$4x - 7y + 2z = 6$ ——— (3)

$(1) \times 2 - (2)$:
$-y - 2z = -10$ ——— (4) **K1**

$(1) \times 4 - (3)$:
$y + 2z = 10$ ——— (5) **K1**

$(4) + (5)$:
$0 = 0$ **J1**

$\therefore$ The system of equations have infinitely many solutions. **B1**

Total: 4

2

Approach 1

$2px + 2y - 6z = 2$ ——— (1)
$qx + 2y - 8z = 7$ ——— (2)
$3x - 2z = 12$ ——— (3)

$(1) - (3) \times 3$:
$2px - 9x + 2y = -34$
$(2p - 9)x + 2y = -34$ **K1**
$y = \dfrac{(9 - 2p)x}{2} - 17$ ——— (4)

$(2) - (3) \times 4$:
$qx - 12x + 2y = 12$
$(q - 12)x + 2y = 12$ **K1**
$y = \dfrac{(12 - q)x}{2} + 6$ ——— (5)

Note that (4) and (5) are equations of a straight line, the two lines will not intersect if and only if they are parallel.

$\dfrac{12 - q}{2} = \dfrac{9 - 2p}{2}$ **B2**
$12 - q = 9 - 2p$
$q = 2p + 3$ **J1**

Approach 2

$2px + 2y - 6z = 2$ ——— (1)
$qx + 2y - 8z = 7$ ——— (2)
$3x - 2z = 12$ ——— (3)

$(1) - (2)$:
$2px - qx + 2z = -5$
$(2p - q)x + 2z = -5$ ——— (4) **K1**

$(3) + (4)$:
$(2p - q)x + 3 = 7$
$(2p - q + 3)x = 7$ ——— (5) **K1**

Note that (5) is a constant function. Since this system of equations have no solutions, we want to make (5) unsolvable.

The only way to do that is if $2p - q + 3 = 0$. As any other values will yield a value of x which satisfies the equation.

$2p - q + 3 = 0$ **B2**
$q = 2p + 3$ **J1**

Total: 5

3

$a + b + c = 10$ —— (1) **B1**

$0.1a + 0.4b + 0.6c = 0.45(10)$
$0.1a + 0.4b + 0.6c = 4.5$ **B1**
$a + 4b + 6c = 45$ —— (2)

$a = 2b$ —— (3) **B1**

By Substitution

$$\begin{aligned} a + b + c &= 10 \\ 2b + b + c &= 10 \\ 3b + c &= 10 \\ c &= 10 - 3b \quad \textbf{K1} \end{aligned}$$

$$\begin{aligned} a + 4b + 6c &= 45 \\ 2b + 4b + 6c &= 45 \\ 6b + 6c &= 45 \quad \textbf{K1} \\ 6b + 6(10 - 3b) &= 45 \\ 6b + 60 - 18b &= 45 \\ -12b &= -15 \\ b &= \frac{5}{4} \quad \textbf{J1} \end{aligned}$$

$$\begin{aligned} a &= 2\left(\frac{5}{4}\right) \\ &= \frac{5}{2} \quad \textbf{J1} \end{aligned}$$

$$\begin{aligned} c &= 10 - 3\left(\frac{5}{4}\right) \\ &= \frac{25}{4} \quad \textbf{J1} \end{aligned}$$

By Elimination

From (3),
$a - 2b = 0$ —— (4)

$(1) - (2)$:
$-3b - 5c = -35$ **K1**
$3b + 5c = 35$ —— (5)

$(1) - (4)$:
$3b + c = 10$ —— (6) **K1**

$(5) - (6)$:
$4c = 25$
$c = \frac{25}{4}$ **J1**

$$\begin{aligned} 3b + c &= 10 \\ 3b + \frac{25}{4} &= 10 \\ b &= \frac{5}{4} \quad \textbf{J1} \end{aligned}$$

$$\begin{aligned} a - 2b &= 0 \\ a - 2\left(\frac{5}{4}\right) &= 0 \\ a &= \frac{5}{2} \quad \textbf{J1} \end{aligned}$$

Total: 8

4

$$\begin{aligned} x + \sqrt{xy} + y &= 13 \\ x + y &= 13 - \sqrt{xy} \end{aligned}$$

$$\begin{aligned} x^2 + xy + y^2 &= 91 \\ x^2 + 2xy + y^2 - xy &= 91 \\ (x + y)^2 - xy &= 91 \quad \textbf{K1} \\ (x + y)^2 &= 91 + xy \\ \left(13 - \sqrt{xy}\right)^2 &= 91 + xy \quad \textbf{Substitution - K1} \\ 169 - 26\sqrt{xy} + xy &= 91 + xy \\ 26\sqrt{xy} &= 78 \\ \sqrt{xy} &= 3 \\ xy &= 9 \\ y &= \frac{9}{x} \end{aligned}$$

$$\begin{aligned} x^2 + xy + y^2 &= 91 \\ x^2 + 9 + \left(\frac{9}{x}\right)^2 &= 91 \quad \textbf{Substitution - K1} \\ x^2 - 82 + \frac{81}{x^2} &= 0 \\ x^4 - 82x^2 + 81 &= 0 \\ \left(x^2 - 81\right)\left(x^2 - 1\right) &= 0 \quad \textbf{Factorisation - K1} \end{aligned}$$

$x^2 - 81 = 0$, $x = \pm 9$ ← **Either - J1** → $x^2 - 1 = 0$, $x = \pm 1$

$y = \frac{9}{\pm 9} = \pm 1$ ← **Both - J1** → $y = \frac{9}{\pm 1} = \pm 9$

For $(9, 1)$
$9 + \sqrt{(9)(1)} + 1 = 13$ (= RHS)

For $(-9, -1)$
$-9 + \sqrt{(-9)(-1)} - 1 = -7$ ($\neq$ RHS)

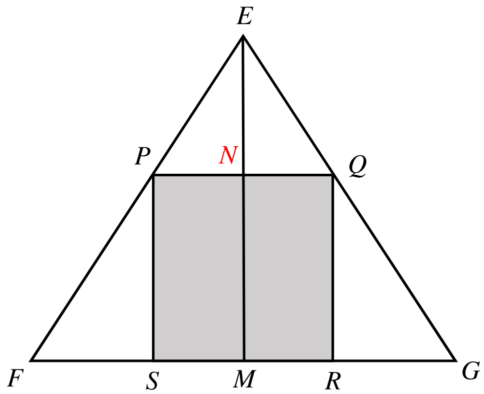
For $(1, 9)$
$1 + \sqrt{(1)(9)} + 9 = 13$ (= RHS)

For $(-1, -9)$
$-1 + \sqrt{(-1)(-9)} - 9 = -7$ ($\neq$ RHS)

$\therefore (9, 1)$ and $(1, 9)$ **Both - J1**

Total: 7

5



Area of rectangle $PQRS = 45$

$2xy = 45$ **B1**

$y = \dfrac{45}{2x}$

Notice that EMG is a right-angled triangle,

$EM^2 + MG^2 = EG^2$

$EM^2 + 8^2 = 17^2$

$EM = 15$ **K1**

Notice that QRG and ENQ are similar triangles,

$$\frac{QR}{RG} = \frac{EN}{NQ}$$

$$\frac{y}{8-x} = \frac{15-y}{x} \quad \textbf{B1}$$

$$xy = (15-y)(8-x)$$

$$xy = 120 - 15x - 8y + xy$$

$$120 - 15x - 8y = 0$$

$$120 - 15x - 8\left(\frac{45}{2x}\right) = 0 \quad \textbf{Substituition - K1}$$

$$120x - 15x^2 - 180 = 0$$

$$15x^2 - 120x + 180 = 0$$

$$x^2 - 8x + 12 = 0$$

$$(x-6)(x-2) = 0 \quad \textbf{Factorisation – K1}$$

$\therefore x = 6, x = 2$

$$y = \frac{45}{2(6)} = \frac{15}{4} \qquad y = \frac{45}{2(2)} = \frac{45}{4}$$

$\therefore x = 6, y = \dfrac{15}{4}$ or $x = 2, y = \dfrac{45}{4}$ **J1**

Total: 6

6

Area of rectangle $PQRS = 168$

$$(6y)(7x) = 168 \quad \textbf{B1}$$

$$42xy = 168$$

$$xy = 4$$

$$y = \frac{4}{x}$$

Radius of semicircle, $r = \dfrac{7x}{2}$

Perimeter $QPRST = 60$

$$\frac{2\pi r}{2} + 7x + 2(6y) = 60 \quad \textbf{B1}$$

$$\pi r + 7x + 12y = 60$$

$$\left(\frac{22}{7}\right)\left(\frac{7x}{2}\right) + 7x + 12\left(\frac{4}{x}\right) = 60 \quad \textbf{Substitution - K1}$$

$$11x + 7x + \frac{48}{x} = 60$$

$$18x^2 + 48 = 60x$$

$$3x^2 - 10x + 8 = 0$$

$$(3x-4)(x-2) = 0 \quad \textbf{Factorisation - K1}$$

$\therefore x = 2, x = \dfrac{4}{3}$

$$y = \frac{4}{2} = 2 \text{ (rejected, } x \neq y\text{)}$$

$$y = \frac{4}{\left(\frac{4}{3}\right)} = 3$$

Both - J1

$\therefore x = \dfrac{4}{3}, y = 3$

$$r = \frac{7x}{2} = \frac{7\left(\frac{4}{3}\right)}{2} = \frac{14}{3}$$

Volume of water $= 15.4$

$$\frac{1}{2}\pi r^2 h = 15.4$$

$$\frac{1}{2}\left(\frac{22}{7}\right)\left(\frac{14}{3}\right)^2 h = 15.4 \quad \textbf{Substitution - K1}$$

$$h = 0.45 \quad \textbf{J1}$$

Total: 7

7	Perimeter of the container $= 1240$ $10x + 2y + x + y = 1240$ **B1** $11x + 3y = 1240$ —— (1) Volume of pillar $= \pi r_1^2 h_1 + 2\pi r_2^2 h_2$ $= \pi \left(\frac{2x}{2}\right)^2 (100) \left(\frac{7y}{10}\pi\right) + 2\pi \left(\frac{2y}{2}\right)^2 (100) \left(\frac{x}{10\pi}\right)$ $= 70x^2y + 20xy^2$ Volume of the container $= (10x)(2y)(x + y)$ $= 20x^2y + 20xy^2$ **Either - B1** Volume of pillar $= 2 \times$ Volume of container $70x^2y + 20xy^2 = 2(20x^2y + 20xy^2)$ **Substitution - K1** $70x^2y + 20xy^2 = 40x^2y + 40xy^2$ $30x^2y - 20xy^2 = 0$ $10xy(3x - 2y) = 0$ Since $xy \neq 0$, $3x - 2y = 0$ —— (2) $x = \frac{2y}{3}$ **By Substitution** $11x + 3y = 1240$ $11\left(\frac{2y}{3}\right) + 3y = 1240$ **Substitution - K1** $31y = 3720$ $y = 120$ $x = \frac{2(120)}{3}$ $= 80$ **Both - J1** **By Elimination** $(1) \times 2 + (2) \times 3$: $31y = 3720$ **K1** $y = 120$ $x = \frac{2(120)}{3}$ $= 80$ **Both - J1** **Total: 6**

INDICES, SURDS AND LOGARITHMS

1

$$\begin{aligned} a &= 2^{165} \times 5^{168} \\ &= 2^{165} \times 5^{165} \times 5^{3} \\ &= (2 \times 5)^{165} \times 125 \\ &= 10^{165} \times 125 \quad \textbf{K1} \end{aligned}$$

The number of digits of 10^n is $n+1$, and the number of digits of $10^n \times b$ is $(n+1)+(m-1) = n+m$, where m is the number of digits of b.

$\therefore a$ has 168 digits. **J1**

Total: 2

2

Let $2^p = 3^q = 36^r = a$

$$2 = a^{\frac{1}{p}}$$
$$3 = a^{\frac{1}{q}}$$
$$36 = a^{\frac{1}{r}}$$

$$2^2 \times 3^2 = 36$$
$$\left(a^{\frac{1}{p}}\right)^2 \times \left(a^{\frac{1}{q}}\right)^2 = a^{\frac{1}{r}} \quad \textbf{Substitution - K1}$$
$$a^{\frac{2}{p}} \times a^{\frac{2}{q}} = a^{\frac{1}{r}}$$
$$\frac{2}{p} + \frac{2}{q} = \frac{1}{r} \quad \textbf{K1}$$
$$\frac{2p+2q}{pq} = \frac{1}{r}$$
$$\frac{2(p+q)}{pq} = \frac{1}{r}$$
$$2r(p+q) = pq \quad \textbf{J1}$$

Total: 3

3

$$\left(x^2 - 5x + 5\right)^{\left(x^2 - 11x + 30\right)} = 1$$

Let $a = x^2 - 5x + 5$
$n = x^2 - 11x + 30$

$a^n = 1$ if $n = 0$ and $a \neq 0$

$x^2 - 11x + 30 = 1$
$(x-6)(x-5) = 1$ **K1**

$x = 6, x = 5$

$a(6) = 6^2 - 5(6) + 5$
$= 11\ (\neq 0)$

$a(5) = 5^2 - 5(5) + 5$
$= 5\ (\neq 0)$

Both - J1

$a^n = 1$ if $a = 1$

$x^2 - 5x + 5 = 1$
$x^2 - 5x + 4 = 0$
$(x-4)(x-1) = 0$ **K1**

$x = 4, x = 1$

$a^n = 1$ if $a = -1$ and n is even

$x^2 - 5x + 5 = -1$
$x^2 - 5x + 6 = 0$
$(x-3)(x-2) = 0$ **K1**

$x = 3, x = 2$

$n(2) = 2^2 - 11(2) + 30$
$= 12$ (even)

$n(3) = 3^2 - 11(3) + 30$
$= 6$ (even)

Both - J1

$\therefore x = 1, 2, 3, 4, 5, 6$ **All - J1**

Total: 6

4	Let $y=\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\cdots}}}$ $y^2=\frac{1}{9(2)}\left(4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\cdots}}\right)$ $y^2=\frac{1}{18}(4-y)$ **B1** $18y^2=4-y$ $18y^2+y-4=0$ $(9y-4)(2y+1)=0$ **Factorisation - K1** $y=\frac{4}{9}, y=-\frac{1}{2}$(rejected) $\therefore y=\frac{4}{9}$ $x=6+\log_{1.5}y$ $=6+\log_{1.5}\frac{4}{9}$ **Substitution - K1** $=6+\log_{1.5}\left(\frac{3}{2}\right)^{-2}$ $=6-2$ $=4$ **J1** **Total: 4**
5	Let $x=\sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}}$ $x^2=\left(\sqrt{3+\sqrt{5}}\right)^2-2\left(\sqrt{3+\sqrt{5}}\right)\left(\sqrt{3-\sqrt{5}}\right)+\left(\sqrt{3-\sqrt{5}}\right)^2$ **K1** $=3+\sqrt{5}-2\sqrt{\left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right)}+3-\sqrt{5}$ $=6-2\sqrt{9-5}$ **K1** $=6-2\sqrt{4}$ $=2$ $x=\sqrt{2}, x=-\sqrt{2}$ (rejected as $x>0$) $\therefore x=\sqrt{2}$ $\log_2\left(\sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}}\right)=\log_2\sqrt{2}$ **Substitution - K1** $=\log_2 2^{\frac{1}{2}}$ $=\frac{1}{2}$ **J1** **Total: 4**

6

$$\log_m p^2 q = x$$
$$\log_m p^2 + \log_m q = x$$
$$2\log_m p + \log_m q = x \quad \text{——} \quad (1)$$

$\longleftarrow$ **Either - B1** $\longrightarrow$

$$\log_m \frac{q}{p} = y$$
$$\log_m q - \log_m p = y \quad \text{——} \quad (2)$$

(1) − (2):

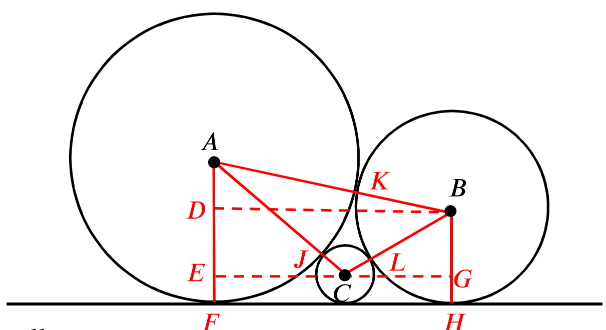
$$3\log_m p = x - y$$
$$\log_m p = \frac{x-y}{3}$$

$$\log_m \frac{q}{p} = y$$
$$\log_m q - \log_m p = y$$
$$\log_m q - \frac{x-y}{3} = y$$ **Substitution - K1**
$$\log_m q = y + \frac{x-y}{3}$$
$$\log_m q = \frac{x+2y}{3}$$

$$\log_m pq = \log_m p + \log_m q$$
$$= \frac{x-y}{3} + \frac{x+2y}{3}$$ **Substitution - K1**
$$= \frac{2x+y}{3}$$ **J1**

Total: 4

7



Recall:
A line drawn from the centre is always perpendicular to the tangent.
When two circles are tangent (externally) to each other, a line drawn through both centre will pass through the common tangential point.

$$CG^2 + BG^2 = CB^2$$
$$CG^2 + (BH - GH)^2 = (CL + LB)^2$$
$$CG^2 + (18 - r)^2 = (18 + r)^2$$ **K1**
$$CG^2 + 324 - 36r + r^2 = 324 + 36r + r^2$$
$$CG^2 = 72r$$
$$CG = \sqrt{72r}$$
$$CG = 6\sqrt{2r}$$

$$CE^2 + AE^2 = AC^2$$
$$CE^2 + (AF - EF)^2 = (AJ + AC)^2$$
$$CE^2 + (30 - r)^2 = (30 + r)^2$$ **K1**
$$CE^2 + 900 - 60r + r^2 = 900 + 60r + r^2$$
$$CE^2 = 120r$$
$$CE = \sqrt{120r}$$
$$CE = 2\sqrt{30r}$$

$$BD^2 + AD^2 = AB^2$$
$$BD^2 + (AF - DF)^2 = (AK + KB)^2$$
$$BD^2 + (30 - 18)^2 = (30 + 18)^2$$ **K1**
$$BD^2 = 2160$$
$$BD = \sqrt{2160}$$
$$BD = 12\sqrt{15}$$

$$BD = CE + CG$$
$$12\sqrt{15} = 2\sqrt{30r} + 6\sqrt{2r}$$ **Substitution - K2**
$$\left(12\sqrt{15}\right)^2 = \left[\sqrt{r}\left(2\sqrt{30} + 6\sqrt{2}\right)\right]^2$$
$$2160 = r\left(120 + 24\sqrt{60} + 72\right)$$
$$r = \frac{2160}{192 + 48\sqrt{15}}$$ **K1**
$$= \frac{45}{4 + \sqrt{15}}$$
$$= \frac{45}{4+\sqrt{15}} \times \frac{4-\sqrt{15}}{4-\sqrt{15}}$$
$$= 180 - 45\sqrt{15}$$ **J1**

Total: 7

8	$\log_x y = \log_y x$ $\log_x y = \dfrac{\log_x x}{\log_x y}$ $(\log_x y)^2 = 1$ **K1** $\log_x y = \pm 1$ (1 is rejected as this causes $x = y$, then $\log_x(x - y)$ is undefined. $\log_x y = -1$ $x^{-1} = y$ $y = \dfrac{1}{x}$ $\log_x(x - y) = \log_y(x + y)$ $\log_x\left(x - \dfrac{1}{x}\right) = \dfrac{\log_x\left(x + \dfrac{1}{x}\right)}{\log_x y}$ **Substitution - K1** $\log_x \dfrac{x^2 - 1}{x} = \dfrac{\log_x \dfrac{x^2+1}{x}}{-1}$ $\log_x \dfrac{x^2 - 1}{x} = \log_x \left(\dfrac{x^2+1}{x}\right)^{-1}$ $\dfrac{x^2 - 1}{x} = \dfrac{x}{x^2+1}$ **K1** $x^4 - 1 = x^2$ $x^4 - x^2 - 1 = 0$ **J1**	**Total: 4**
9	(*a*)	$\text{SOR} = -\dfrac{b}{a}$ $\log_2 a + \log_2 b + \log_2 c + di - di = -\left(-\dfrac{3}{1}\right)$ **K1** $\log_2 abc = 3$ $abc = 2^3$ $abc = 8$ **J1**
	(*b*)	$\dfrac{b}{a} = \dfrac{c}{b}$ $b^2 = ac$ $abc = 8$ $acb = 8$ $b^2 b = 8$ **Substitution - K1** $b^3 = 8$ $b = 2$ $\log_2 b = \log_2 2$ $= 1$ **J1**

(c)

$$b^2 = ac$$
$$2^2 = ac$$
$$c = \frac{4}{a}$$

$$\text{POR} = -\frac{z}{a}$$
$$\log_2 a \times \log_2 b \times \log_2 c \times di \times -di = -\frac{q}{1}$$
$$\log_2 a \times 1 \times \log_2 \frac{4}{a} \times \left(-d^2 i^2\right) = -8d^2$$ **Substitution - K1**
$$\log_2 a\,(\log_2 4 - \log_2 a) \times -1\left(d^2\right)(-1) = -8d^2$$
$$2\log_2 a - (\log_2 a)^2 = -8$$
$$(\log_2 a)^2 - 2\log_2 a - 8 = 0$$
$$(\log_2 a - 4)(\log_2 a + 2) = 0$$ **Factorisation - K1**

$$\log_2 a = 4, \log_2 a = -2$$
$$\therefore a = 16, a = \frac{1}{4}$$

$$c = \frac{4}{a}$$
$$c = \frac{4}{16}$$
$$= \frac{1}{4}$$

← Both - J1 →

$$c = \frac{4}{a}$$
$$c = \frac{4}{\left(\frac{1}{4}\right)}$$
$$= 16$$

$$\log_2 c = \log_2 \frac{1}{4}$$
$$= -2$$

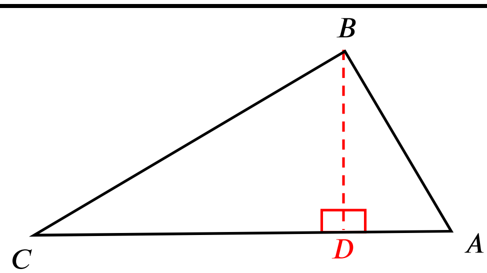
$$\log_2 c = \log_2 16$$
$$= 4$$

$\therefore$ The two other real roots are -2 and 4.

B1

Total: 8

10



$$\sin 60^\circ = \frac{BD}{AB}$$
$$\frac{\sqrt{3}}{2} = \frac{BD}{3\sqrt{3}}$$ **B1**
$$BD = \frac{9}{2}$$

$$\tan 60^\circ = \frac{BD}{AD}$$
$$\sqrt{3} = \frac{\left(\frac{9}{2}\right)}{AD}$$ **Substitution - K1**
$$AD = \frac{9}{2\sqrt{3}}$$
$$= \frac{9}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{3\sqrt{3}}{2}$$

$$CD = AC - AD$$
$$= 4\sqrt{3} - \frac{3\sqrt{3}}{2}$$ **Substitution - K1**
$$= \frac{5\sqrt{3}}{2}$$

$$BC^2 = CD^2 + BD^2$$
$$= \left(\frac{5\sqrt{3}}{2}\right)^2 + \left(\frac{9}{2}\right)^2$$ **Substitution - K1**
$$= \frac{75}{4} + \frac{81}{4}$$
$$= 39$$
$$BC = \sqrt{39}$$ **J1**

Total: 5

11	$\log_{30} 8 = \log_{30} 2^3$ $= 3\log_{30} 2$ **K1** $= 3\log_{30}\left(\frac{30}{15}\right)$ $= 3(\log_{30} 30 - \log_{30} 15)$ **K1** $= 3(1 - \log_{30} 3 \times 5)$ $= 3 - 3(\log_{30} 3 + \log_{30} 5)$ $= 3 - 3\log_{30} 3 - 3\log_{30} 5$ **J1** **Total: 3**

PROGRESSIONS

1

$$2\cos\theta - \sin\theta = 2\sin\theta - 2\cos\theta \quad \textbf{B1}$$
$$4\cos\theta = 3\sin\theta$$
$$\frac{\sin\theta}{\cos\theta} = \frac{4}{3}$$
$$\tan\theta = \frac{4}{3}$$
$$\theta = \tan^{-1}\frac{4}{3}$$
$$= 53.13^\circ \quad \textbf{J1}$$

Total: 2

2

$$\log_{10} x + \log_{10} x^2 + \log_{10} x^3 + \log_{10} x^4 + \cdots + \log_{10} x^n = n(n+1)$$
$$\log_{10}(x)\left(x^2\right)\left(x^3\right)\ldots\left(x^n\right) = n(n+1)$$
$$\log_{10} x^{1+2+3+\ldots+n} = n(n+1) \quad \textbf{B1}$$
$$\log_{10} x^{\left(\frac{n}{2}\right)(1+n)} = n(n+1)$$
$$x^{\frac{n(n+1)}{2}} = 10^{n(n+1)}$$
$$\left(x^{\frac{1}{2}}\right)^{n(n+1)} = 10^{n(n+1)} \quad \textbf{K1}$$
$$x^{\frac{1}{2}} = 10$$
$$x = 100 \quad \textbf{J1}$$

Total: 3

3

$$\frac{b}{c} = \frac{a}{b}$$
$$b^2 = ac$$

$$ax^2 + 2bx + c = 0$$

$$x = \frac{-2b \pm \sqrt{(2b)^2 - 4ac}}{2a} \quad \textbf{Substitution - K1}$$
$$= \frac{-2b \pm \sqrt{4b^2 - 4b^2}}{2a}$$
$$= \frac{-2b \pm 0}{2a}$$
$$= -\frac{b}{a}$$

$$px^2 + 2qx + r = 0$$
$$p\left(-\frac{b}{a}\right)^2 + 2q\left(-\frac{b}{a}\right) + r = 0 \quad \textbf{Substitution - K1}$$
$$p\left(\frac{b^2}{a^2}\right) - 2q\left(\frac{b}{a}\right) + r = 0$$
$$\frac{pac}{a^2} - \frac{2qb}{a} + r = 0 \quad \textbf{Substitution - K1}$$
$$\frac{pc}{a} - \frac{2qb}{a} + r = 0$$
$$pc - 2qb + ra = 0$$
$$pc - qb - qb + ra = 0$$
$$pc - qb = qb - ra \quad \text{—— (1)}$$

$(1) \div ac$:
$$\frac{pc}{ac} - \frac{qb}{ac} = \frac{qb}{ac} - \frac{ra}{ac} \quad \textbf{B1}$$
$$\frac{pc}{ac} - \frac{qb}{b^2} = \frac{qb}{b^2} - \frac{ra}{ac}$$
$$\frac{p}{a} - \frac{q}{b} = \frac{q}{b} - \frac{r}{c}$$

Since $\frac{p}{a} - \frac{q}{b} = \frac{q}{b} - \frac{r}{c}$.
Therefore $\frac{p}{a}, \frac{q}{b}$ and $\frac{r}{c}$ forms an arithmetic progression.

B1

Total: 5

4

Multiples of 3: $3, 6, 9, \ldots, 999$
Multiples of 5: $5, 10, 15, \ldots, 995$
Multiples of 15: $15, 30, 45, \ldots, 990$

$999 = 3 + (n_1 - 1)3$
$n_1 = 333$ ← **Both - J1** → $995 = 5 + (n_2 - 1)5$
$n_2 = 199$

$990 = 15 + (n_3 - 1)15$
$n_3 = 66$ **J1**

$$\text{Sum} = \frac{333}{2}(3 + 999) + \frac{199}{2}(5 + 995) - \frac{66}{2}(15 + 990)$$ **Substitution - K1**

$$= 233168$$ **J1**

Total: 4

5

The progression consists of two arithmetic progression.

Progression 1: $3, 10, 17, 24, \ldots$

$a_1 = 3$
$d_1 = 10 - 3$
$= 7$

$T_n = a_1 + (n_1 - 1)d_1$
$138 = 3 + (n - 1)7$
$n_1 = 20.29$ (rejected)

$T_n = a_1 + (n_1 - 1)d_1$
$143 = 3 + (n - 1)7$
$n_1 = 21$ **J1**

Progression 2: $5, 12, 19, \ldots$

$a_2 = 5$
$d_2 = 12 - 5$
$= 7$

$T_n = a_2 + (n_2 - 1)d_2$
$138 = 5 + (n_2 - 1)7$
$n_2 = 20$ **J1**

$T_n = a_2 + (n_2 - 1)d_2$
$143 = 5 + (n - 1)7$
$n_2 = 20.71$ (rejected)

Thus, 138 is the last term of the second progression, and 143 is the last term of the first progression.

$$\text{Sum} = S_{n1} + S_{n2}$$
$$= \frac{21}{2}(3 + 143) + \frac{20}{2}(5 + 138)$$ **Substitution - K1**
$$= 2963$$ **J1**

Total: 4

6

(*a*)

$a_P = 10$
$d_P = 2$
$n = t$

$$S_P = \frac{n}{2}\left[2a_P + (n - 1)d_P\right]$$
$$= \frac{t}{2}[2(10) + (t - 1)2]$$
$$= t(10 + t - 1)$$
$$= 9t + t^2$$ **J1**

$a_Q = 8$
$d_Q = 0$
$n = t$

$$S_Q = \frac{n}{2}\left[2a_Q + (n - 1)d_Q\right]$$
$$= \frac{t}{2}[2(8) + (t - 1)0]$$
$$= 8t$$ **J1**

$$S_P + S_Q = 920$$
$$9t + t^2 + 8t = 920$$ **Substitution - K1**
$$t^2 + 17t - 920 = 0$$
$$(t - 23)(t + 40) = 0$$ **Factorisation - K1**
$$t = 23, t = -40 \text{ (rejected)}$$
$$t = 23$$ **J1**

(*b*)

$$S_Q = 920$$
$$8t = 920$$
$$t = 115$$

$$S_P = 9(115) + 115^2$$ **Substitution - K1**
$$= 14260 \text{ m}$$ **J1**

Total: 7

7 (*a*) i) The number of integers of the sequence forms a geometric progression: 1, 2, 4, 8, …

First term = 1

$$\text{Common ratio} = \frac{4}{2} = 2$$

$$\text{Number of integers in the } n^{th} \text{ bracket} = (1)\left(2^{n-1}\right) = 2^{n-1} \quad \textbf{J1}$$

ii) The first term of each bracket, a, forms a geometric progression: 1, 2, 4, 8, …

First term = 1

$$\text{Common ratio} = \frac{4}{2} = 2$$

$$a = (1)\left(2^{n-1}\right) = 2^{n-1} \quad \textbf{J1}$$

iii) In each bracket, the integers form an arithmetic progression, $a, a+1, a+2, \dots, l$, where l is the last term in each bracket.

First term $= a = 2^{n-1}$

Common difference, $d = a + 1 - a = 1$

Number of terms $= n = 2^{n-1}$

$$\begin{aligned} l &= a + (n-1)d \\ &= 2^{n-1} + \left(2^{n-1} - 1\right)(1) \quad \textbf{Substitution - K1} \\ &= 2\left(2^{n-1}\right) - 1 \\ &= 2^n - 1 \quad \textbf{J1} \end{aligned}$$

(*b*) In each bracket, the integers form an arithmetic progression: $a, a+1, a+2, \dots, l$, where l is the last term in each bracket.

First term $= a = 2^{n-1}$

Last term $= l = 2^n - 1$

Number of terms $= n = 2^{n-1}$

$$\begin{aligned} S_n &= \frac{n}{2}(a + l) \\ &= \frac{2^{n-1}}{2}\left(2^{n-1} + 2^n - 1\right) \quad \textbf{Substitution - K1} \\ &= 2^{n-2}\left[2^{n-1} + 2\left(2^{n-1}\right) - 1\right] \\ &= 2^{n-2}\left[3\left(2^{n-1} - 1\right)\right] \quad \textbf{J1} \end{aligned}$$

Total: 6

8

$$\text{Let } \frac{1}{5} + \frac{2}{25} + \frac{3}{125} + \frac{4}{625} + \cdots = S_n \quad \text{—— (1)}$$

$$1 + \frac{2}{5} + \frac{3}{25} + \frac{4}{125} + \cdots = 5S_n \quad \text{—— (2)}$$

(2) − (1):

$$1 + \frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \cdots = 4S_n \quad \textbf{K1}$$

$$a = 1 \qquad r = \frac{1}{5} \div 1 = \frac{1}{5}$$

$$S_\infty = 4S_n$$

$$\frac{1}{\frac{1}{5} - 1} = 4S_n \quad \textbf{Substitution - K1}$$

$$\frac{5}{4} = 4S_n$$

$$S_n = \frac{5}{16} \quad \textbf{J1}$$

$$\frac{1}{5} + \frac{2}{25} + \frac{3}{125} + \frac{4}{625} + \cdots = S_n = \frac{5}{16}$$

Total: 3

9	(*a*)	$a = 15$ $d = \frac{8}{2} = 4$ $131\pi = \frac{2\pi r}{2}$ $r = 131$ $T_n = 131$ $15 + (n-1)4 = 131$ **Substitution - K1** $n = 30$ $\therefore 30^{th}$ semicircle. **J1**
	(*b*)	Radius of the final semicircle $= OP - 4$ $= 211$ cm $T_n = a + (n-1)d$ $211 = 15 + (n-1)4$ **Substitution - K1** $n = 50$ **J1** The arc length of each circle $= \pi r$ Thus, the arc length follows an arithmetic progression: $15\pi, 19\pi, 23\pi, \ldots$ $a = 15\pi$ $d = 19\pi - 15\pi = 4\pi$ $S_n = \frac{50}{2}[2(15\pi) + (50-1)(4\pi)]$ **Substitution - K1** $= 50(15\pi + 98\pi)$ $= 5650\pi$ cm **J1** **Total: 6**
10	(*a*)	Let $T_{n+1} - T_n = T_n - T_{n-1} = d$ Then $T_n - T_{n+1} = T_{n-1} - T_n = -d$ $\frac{U_{n+1}}{U_n} - \frac{U_n}{U_{n-1}} = \frac{2^{3(4-T_{n+1})}}{2^{3(4-T_n)}} - \frac{2^{3(4-T_n)}}{2^{3(4-T_{n-1})}}$ **Substitution - K1** $= \frac{8^{(4-T_{n+1})}}{8^{(4-T_n)}} - \frac{8^{(4-T_n)}}{8^{(4-T_{n-1})}}$ $= 8^{4-T_{n+1}-(4-T_n)} - 8^{4-T_n-(4-T_{n-1})}$ **K1** $= 8^{T_n - T_{n+1}} - 8^{T_{n-1} - T_n}$ $= 8^{-d} - 8^{-d}$ **Substitution - K1** $= 0$ $\frac{U_{n+1}}{U_n} - \frac{U_n}{U_{n-1}} = 0$ $\frac{U_{n+1}}{U_n} = \frac{U_n}{U_{n-1}}$, thus U_n is the n^{th} term of a geometric progression. **B1**
	(*b*)	$T_n - T_{n-1} = 2$ $T_{n-1} - T_n = -2$ $\frac{U_n}{U_{n-1}} = 8^{T_{n-1} - T_n}$ $r = 8^{-2}$ **Substitution - K1** $= \frac{1}{64}$ **J1** **Total: 6**

11		$\dfrac{T_3}{T_1} = \dfrac{ar^{3-1}}{a}$ $R_1 = r^2$ Sum of odd terms $= \dfrac{27}{4}$ $\dfrac{T_1}{1 - R_1} = \dfrac{27}{4}$ $\dfrac{a}{1 - r^2} = \dfrac{27}{4}$ —— (1) ← **Either - K1** → $\dfrac{ar}{1 - r^2} = \dfrac{9}{4}$ —— (2) $\dfrac{T_4}{T_2} = \dfrac{ar^{4-1}}{ar^{2-1}}$ $R_2 = r^2$ Sum of even terms $= \dfrac{9}{4}$ $\dfrac{T_2}{1 - R_2} = \dfrac{9}{4}$ (2) ÷ (1): $r = \dfrac{1}{3}$ **J1** $\dfrac{a}{1 - \left(\frac{1}{3}\right)^2} = \dfrac{27}{4}$ $a = 6$ **J1** **Total: 3**
12	(*a*)	$a = 200 \times 12$ $= 2400$ $d = (250 - 200) \times 12$ $= 600$ $n = 2004 - 2000 + 1$ $= 5$ $S_n = \dfrac{n}{2}[2a + (n-1)d]$ $= \dfrac{5}{2}[(2(2400) + (5-1)(600)]$ **Substitution - K1** $= 18000$ Total money $= 18000 \times 1.05$ **Substitution - K1** $=$ RM18900 **J1**
	(*b*)	$n = 2017 - 2000 + 1$ $= 18$ $S_{10} = \dfrac{10}{2}[2(2400) + (10-1)(600)]$ **Substitution - K1** $= 51000$ $S_{18} = 51000(1.1)^{18-10}$ **Substitution - K1** $=$ RM 109323.03 Average annual interest rate $= \dfrac{109323.03 - 51000}{51000} \times 100\% \div 18$ **Substitution - K1** $= 6.35\%$ **J1** **Total: 7**